Physics Challenges for Teachers and Students

Solutions to September 2003 Challenges

The Road Rage

Challenge: Two cars begin to move toward each other simultaneously along a straight road. Car 1 starts from point A at a speed v_1; car 2 starts at point B at a speed v_2. The acceleration of car 1 is a_1; it is directed toward A. The acceleration of car 2 is a_2; it is directed toward B. In the process of motion, the cars meet twice; the time interval between the meetings is t. Find the distance between A and B.

Solution: It helps to switch the reference frame; for instance, let us assume that A is at rest. Let A be located at $x = 0$. Now, the velocity and acceleration of the car from B have values of $-(v_1 + v_2)$, $(a_1 + a_2)$ respectively.

The times of intersection of A and B are given by $0 = x_B - (v_1 + v_2)t + \frac{1}{2}(a_1 + a_2)t^2$ and by solving the quadratic equation:

$$t_{int} = \frac{(v_1 + v_2) \pm \sqrt{(v_1 + v_2)^2 - 2x_B(a_1 + a_2)}}{(a_1 + a_2)}.$$

The time interval between crossings is therefore

$$t = \frac{2\sqrt{(v_1 + v_2)^2 - 2x_B(a_1 + a_2)}}{(a_1 + a_2)}.$$

Rearranging gives

$$\frac{1}{4}(a_1 + a_2)^2 t^2 = (v_1 + v_2)^2 - 2x_B(a_1 + a_2).$$

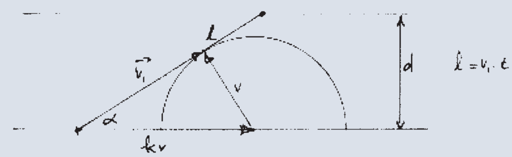
Hence $x_B = \dfrac{(v_1 + v_2)^2}{2(a_1 + a_2)} - \dfrac{1}{8}(a_1 + a_2)t^2.$

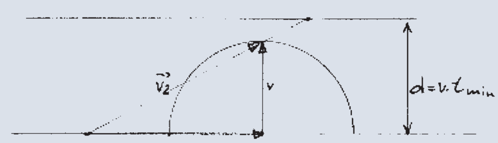
(Contributed by Michael C. Faleski, Delta College, Midland, MI)

Riverdance

Challenge: A child in a boat needs to cross the river. The speed of the current in the river is k times greater than the speed of the boat in still water. If a child crosses the river in such a way as to minimize the lateral displacement, it takes time t to cross. What is the minimum time required to cross the river?

Solution: The two sketches below show two possible directions for the resultant velocity of the child (v_1 at the top sketch, v_2 at the bottom one). When the child chooses the direction $\vec{v_1}$ (see top sketch), angle α is maximized, and she gets the minimum lateral displacement.





From the top sketch:

$$\sin\alpha = 1/k \text{ and } \cos\alpha = \sqrt{1 - \frac{1}{k^2}} = \frac{1}{k}\sqrt{k^2 - 1}.$$

For the minimum time we find:

$d = v\, t_{min}$.

For the minimum displacement, the distance traveled (let us call it l) is given by

$d = l \sin\alpha$,

and the time required to cross with minimum lateral displacement is given by

$(kv\cos\alpha)\, t = l$ (because $v_1 = kv\cos\alpha$).

Combining these equations gives:

$$t_{min} = k \sin\alpha \cos\alpha$$

or

$$t_{min} = t \cdot \frac{\sqrt{k^2 - 1}}{k}.$$

(Contributed by Hubert N. Biezeveld, Zwaag, The Netherlands)

The Flight of the Bumblebee

Challenge: A rock is launched upward at 45°. A bumblebee follows the trajectory of the rock at a *constant* speed equal to the *initial* speed of the rock. What is the acceleration of the bumblebee at the top point of the trajectory? For the rock, neglect the air resistance.

Solution: At the top of its trajectory the ball's velocity is $v/\sqrt{2}$, where v is the ball's initial speed. Its acceleration is just g, perpendicular to the velocity, so the radius of curvature of the ball's trajectory at that instant is

$$R = \frac{v^2}{2g}. \qquad (19)$$

The bumblebee follows the rock's trajectory but at a constant speed of v, so at the top of the trajectory its speed is still v, and the radius of curvature of its trajectory is the same as that of the ball. So its acceleration at that point is

$$a = \frac{v^2}{R} = 2g.$$

(Contributed by Hugh Haskell, NC School of Science and Mathematics, Cary, NC)

Solutions:

Many other readers also sent us correct solutions to the September *Challenges*. We would like to recognize the following contributors:

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Note to contributors:

We appreciate your submissions and hope to receive more solutions in the future.
Note to contributors: as the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– Please email the solutions as Word files;
– Please name the file as "Dec03HSimpson" if—for instance—your name is Homer Simpson, and you are sending the solutions to December 2003 Challenges;
– Please state your name, hometown, and professional affiliation in the file, not only in the email message.

Many thanks!

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