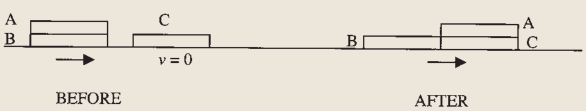
Physics Challenges for Teachers and Students

Solutions to November 2003 Challenges

The Board Game

Challenge: Board A is placed on board B as shown. Both boards slide, without moving with respect to each other, along a frictionless horizontal surface at a speed *v*. Board B hits a resting board C "head-on." After the collision, boards B and C move together, and board A slides on top of board C and stops its motion relative to C in the position shown on the diagram. What is the length of each board? All three boards have the same mass, size, and shape. It is known there is no friction between boards A and B; the coefficient of kinetic friction between boards A and C is μ_k.



Solution: First consider the situation immediately after the perfectly inelastic collision between B and C but before A has begun to slip onto C; label this as the initial configuration "i." Clearly $\upsilon_{Ai} = \upsilon$ and $\upsilon_{Bi} = \upsilon_{Ci} = \upsilon/2$ by momentum conservation. On the other hand, in the end (labeled "f") all three masses have the same final speed, so that $\upsilon_{Af} = \upsilon_{Bf} = \upsilon_{Cf} = 2\upsilon/3$ by again applying momentum conservation. The sum of the kinetic energies of the boards thus changes by

$$\Delta K = (K_{Af} + K_{Bf} + K_{Cf}) - (K_{Ai} + K_{Bi} + K_{Ci})$$

$$= -\frac{1}{12}mv^2, \quad (1)$$

where m is the mass of each board. This net loss in mechanical energy is the result of the nonconservative "internal work" done by friction,

$$W_{NC} = -\int f\, dx_A + \int f\, dx_C = -\int_0^L f\, d(x_A - x_C), \quad (2)$$

where the equal and opposite internal force of friction between blocks A and C is given by

$$f = \mu_k N = \mu_k mg \frac{x_A - x_C}{L}, \quad (3)$$

with L the length of each board. Here, x_A and x_C are the rightward displacements of blocks A and C from their initial positions; the ratio $(x_A - x_C)/L$ thus gives the fraction of block A, which is on top of block C, and hence the ratio of the normal force N of A on C to the weight mg of A. Substitute Eq. (3) into Eq. (2) to find

$$W_{NC} = \frac{\mu_k mg}{L}\frac{L^2}{2}. \quad (4)$$

It is worth pausing to note that it would have been *extremely* difficult to attempt to separately calculate the two frictional work terms in the middle expression in Eq. (2)! Finally equate the right-hand sides of Eqs. (1) and (4) to obtain the answer,

$$L = \frac{\upsilon^2}{6\mu_k g}. \quad (5)$$

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

A Pit Pull

Challenge: A small bucket rests at the bottom of a pit that has a depth *h*. A winch with an elastic cord is used to lift the bucket out of the pit. One end of the cord is attached to the bucket; the other end is attached to the drum of the winch. There is a mark on the cord at the height 0.8*h* from the bottom of the pit. The cord is vertical and relaxed but taut. The winch begins to rotate slowly. It is noticed that the bucket loses contact with the ground just as the mark on the cord reaches the drum. How much work is done by that moment?

(**Column Editor's note:** The mass of the bucket should have been given; most readers did assume that in their solutions. We apologize for that omission.)

Solution: Since the gravitational potential energy of the bucket does not change, the entire work done increases the elastic potential energy of the cord. The segment of cord between the bucket and the mark increases its length from $0.8h$ to h. Therefore, if the cord's elastic behavior is described by Hooke's equation, the work done on stretching that segment of the cord is

$$kx^2\backslash 2 = (mg/x)(x^2)/2 = mg(0.2h)/2 = 0.1mg.$$

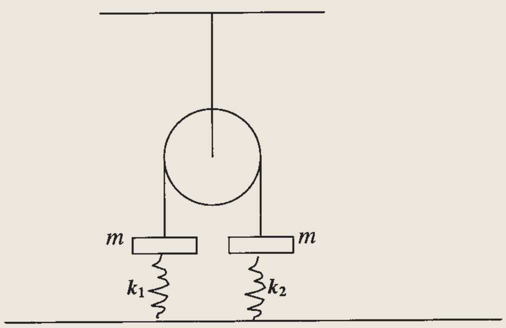
(Note that the bucket loses contact with the ground when $kx = mg$.) But the part of the cord that winds around the drum is also stretched. The work done stretching it depends on the friction between the cord and drum.

If there is no friction, then the tension of that part of the cord is always equal to the tension of the vertical part, and the elastic potential energy is uniformly distributed along the entire cord. The total stored energy will then be $(1/0.8 = 1.25)$ times the energy found above, or $0.125\ mgh$.

If there is substantial friction between the drum and the cord, the tension will not be uniform in the part of the cord that is wound around the drum. The average tension in that part would then be roughly $mg/2$, and the potential energy in that part will be roughly $0.0125\ mgh$. The total work done then will be roughly $0.1125\ mgh$.

In reality, there will be some rubbing between the cord and the drum as the drum turns, causing some heating. The average tension in the wound-up cord will then be somewhat greater than $mg/2$, so the total work done by the engine will be closer to $0.125\ mgh$ than to $0.1125\ mgh$.

(Contributed by Art Hovey, Milford, CT)



A Spring Fling

Challenge: On the diagram, two blocks of equal mass are connected by an ideal string. The values of m, k_1, and k_2 are given ($k_1 > k_2$). Initially, both springs are relaxed. Then the left block is slowly pulled down a distance x and released. Find the acceleration of each block immediately after the release. Find *all* possible answers.

Solution: Let T be the tension in the ideal string and a be the acceleration of the blocks at the instant of release. For the block on the left, the upward acceleration may be found from

$$T + k_1x - mg = ma.$$

For the block on the right, the downward acceleration may be found from

$$k_2x + mg - T = ma.$$

Adding the equations gives the acceleration of the blocks as

$$a = (k_1 + k_2)x/(2m).$$

However, substracting the equations gives

$$T = mg - (k_1 - k_2)x/2.$$

But a negative T would indicate compression of the ideal string. So $a = (k_1 + k_2)x/(2m)$ only if $k_1 < k_2 + 2mg/x$. If $k_1 > k_2 + 2mg/x$, $T = 0$ and the blocks accelerate independently:

$$a_1 = k_1x/m - g \quad \text{and} \quad a_2 = k_2x/m + g.$$

(Contributed by John F. Goehl Jr., Barry University, Miami Shores, FL)

Several other readers also sent us correct solutions to the November Challenges. We would like to recognize the following contributors:

Jia He, student (St. John's Prep School, Collegeville, MN)

Eugene P. Mosca (U.S. Naval Academy, Annapolis, MD)

We appreciate your submissions and hope to receive more solutions in the future.

Note to contributors:

As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– please email the solutions as Word files;

– please name the file as "Feb04LSimpson" if — for instance — your name is Lisa Simpson, and you are sending the solutions to February 2004 Challenges;

– please state your name, hometown and professional affiliation in the *file*, not only in the email message.

Many thanks!

Please send correspondence to:

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