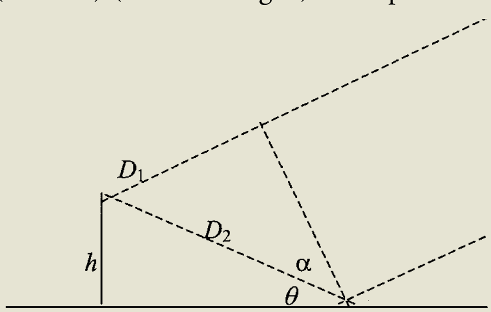
Physics Challenges for Teachers and Students

Solutions for May 2003 Challenges

Lake Placid

Challenge: A radio receiver is set up on a mast in the middle of a calm lake to track the radio signal from a satellite orbiting the Earth. As the satellite rises above the horizon, the intensity of the signal varies periodically. The intensity is at a maximum when the satellite is $\theta_1 = 3°$ above the horizon and then again at $\theta_2 = 6°$ above the horizon. What is the wavelength λ of the satellite signal? The receiver is h = 4.0 m above the lake surface.

Solution: Distance from reflection point to top of mast = $D_2 = h/\sin\theta = h/\theta$ (for small angles). Corresponding distance on direct path from source to receiver: $D_1 = D_2 \sin\alpha$. But $\alpha = (\pi - \pi/2 - 2\theta)$, so $D_1 = D_2 \cos 2\theta = (h/\theta)(1 - 2\theta^2)$ (for small angles). The path



difference is then $D_2 - D_1 = 2h\theta$.
As the satellite rises from 3 to 6 degrees above the horizon, that path difference changes by one wavelength, so $\lambda = 2h(6 - 3)(\pi/180)$ = 0.42 m.

(Contributed by Art Hovey, Milford, CT)

•***Column Editor's comment***: This answer, obtained by several readers, assumes no phase change upon reflection.

Ultimate Excitement

Challenge: The amplitude of the electric field in an electromagnetic wave of frequency $\omega = 2.0 \times 10^{16}\ s^{-1}$ changes with time as $E(t) = k\,(1 + \cos \Omega t)$, where k is a constant and $\Omega = 1.8 \times 10^{15}\ s^{-1}$. Would such a wave cause ionization of hydrogen atoms? If yes, what is the energy of the ejected electrons E_c? Assume that atoms absorb light as photons. The ionization energy of hydrogen gas is E_i = 13.6 eV. The Planck constant $\hbar$ = 1.05 J × s.

Solution: The expression for the electric field can be obtained as

$$E = k(1 + \cos \Omega t)\cos \omega t$$
$$= k \cos \omega t + \frac{1}{2} k \cos(\omega - \Omega)t + \frac{1}{2} k \cos(\omega + \Omega)t,$$

where the product of the two cosines is expanded using a standard trigonometric identity. These three terms correspond to photons of energies hω, h$(\omega - \Omega)$, and h$(\omega + \Omega)$. The latter exceeds the ionization energy by 0.7 eV. That difference equals the ejected electron energy.

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

Home Plate

A metal plate is exposed to light with wavelength λ. It is observed that electrons are ejected from the surface of the plate. When a retarding electric field E is imposed, no electron can move away from the plate farther than a certain distance d. Find the threshold wavelength λ_0 for the material of the plate.

Solution: The kinetic energy of the electrons immediately upon ejection is the difference between the energy of the incident photon and the threshold energy:

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}.$$

This kinetic energy of the ejected electron is converted into potential energy, $U = eEd$, as the electrons move in the direction of the retarding electric field, assuming it to be approximately uniform. Therefore,

$$K = Eed$$

and

$$\lambda_0 = \left(\frac{1}{\lambda} - \frac{eEd}{hc}\right)^{-1}.$$

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

Other Solutions

Many other readers also sent us their solutions to the May Challenges. We would like to recognize the following contributors:

J. J. Carr (*Webster, NY*)

David A. Cornell (*Principia College, Elsah, IL*)

John F. Goehl Jr. (*Barry University, Miami Shores, FL*)

Robbie F. Kouri (*Our Lady of the Lake University, San Antonio, TX*)

Göran Norlén (*Lund, Sweden*)

Adam Plana (*Wheatley School, Old Westbury, NY*)

Reza Vafabakhsh, student (*IASBS, Zanjan, Iran*)

Clarke Wellborn and his students at Brevard College (*Brevard, NC*)

We appreciate your submissions and hope to receive more solutions (and, maybe, some problems, too) in the future.

Note to contributors: In order to facilitate the process more efficiently, please observe these submission guidelines:

– email the solutions as Word files;
– name the file "September03BSimpson" if — for instance — your name is Bart Simpson, and you are sending the solutions to September 2003 Challenges;
– state your name, hometown, and professional affiliation in the file, not only in the email message.

Many thanks!

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