Physics Challenges for Teachers and Students

Here we present solutions to the May 2002 *Challenges.*

Show Me the Money

The only way that moving the object a distance equal to the focal length will produce a new image of the same size is if the motion occurs from outside the focal point to inside the focal point, or vice versa. Let d_o and D_o be the two object distances where the magnification of the lens is the same, and d_i and D_i be the corresponding image distances. Distances d_o and D_o are related by

$$D_o - d_o = f, \tag{1}$$

where f is the focal length of the lens. By the construction here, $d_o < D_o$ and D_o lies outside the focal point while d_o lies inside it. For the outside point, solving the lens equation for D_i gives

$$D_i = fD_o / (D_o - f) \tag{2}$$

and for d_i gives

$$D_i = fd_o / (f - d_o), \tag{3}$$

where we have taken d_i to be numerically positive and incorporated the location of the virtual image on the "negative" side of the lens directly into the lens equation for the virtual image. This will allow us to write the magnification as positive without resorting to absolute values. Since magnification can be written

$$m = \frac{D_i}{D_o}, \tag{4}$$

we find from Eq. (2) that

$$m = f / (D_o - f) \tag{5}$$

and similarly, using Eq. (3),

$$m = f / (f - d_o)\ . \tag{6}$$

Setting Eqs. (5) and (6) equal to each other and substituting for d_o from Eq. (1) gives

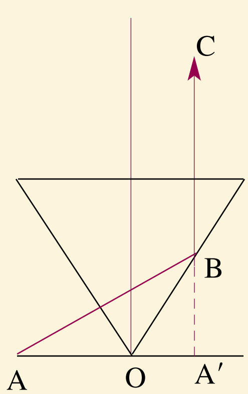
$$D_o = 3f/2\ . \tag{7}$$

Putting Eq. (7) into Eq. (5) gives $m = 2$, and since the diameter of the coin was 4 cm, the diameter of its image is 8 cm.

(*Contributed by Hugh Haskell, North Carolina School of Science and Mathematics, Durham, NC*)

The Cone Artist

An observer far above the cone must see light rays coming from the edge of the plate, which pass though the glass and emerge from the top side of the cone roughly parallel to the axis of



the cone. For glass the refractive index is usually around 1.5, so the critical angle is about 42°. Rays in glass coming from a glass-air boundary at an angle greater than that critical value can only do so by reflection.

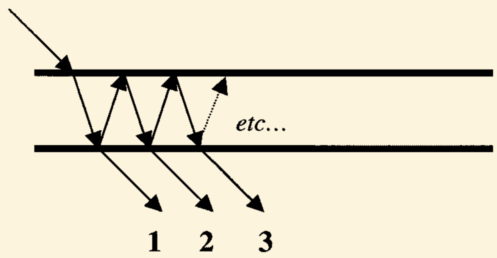
The angle between ray BC and the normal at B is 60°, which is far greater than the estimated critical angle. Therefore, ray BC must be the result of reflection off the inner conical surface. Ray ABC appears to be coming from A′, so the apparent radius of the plate is r = OA′ while the real radius is R = OA. All of the angles are 30, 60, or 90°. Triangle ABO splits into two right triangles that are congruent to triangle OA′B, so $r = R/2$.

(*Contbuted by Art Hovey, Amity Regional Senior High School, Woodbridge, CT*)

Through the Looking Glass

Ray 1 carries $(1-k)^2$ of the beam's energy;
Ray 2 carries $k^2(1-k)^2$ of the beam's energy;
Ray 3 carries $k^4(1-k)^2$ of the beam's energy;
etc.
The total fraction of transmitted energy is

$$(1-k)^2 + k^2(1-k)^2 + k^4(1-k)^2 + \ldots.$$
$$= (1-k)^2(1 + k^2 + k^4 + \ldots) = (1-k)^2/(1-k^2)$$
$$= (1-k)/(1+k) = 7/13 = 53.8\%.$$



(*Contributed by Gerald Hite, A&M University at Galveston, Galveston, TX*)