Physics Challenges for Teachers and Students

Solutions for March 2003 Challenges

Ups and Downs Under Pressure

Challenge: The bottoms of two vertical cylinders of different cross sections are connected by a thin pipe. Each cylinder contains gas at constant temperature and is covered by a movable piston. The mass of one piston is m_1 = 1.0 kg, the other m_2 = 2.0 kg. Initially, the pistons are at the same height h = 0.40 m. What would be the difference in the heights of the pistons (H) if an extra load of m = 1.0 kg is placed on the lighter piston? Assume the entire arrangement is placed in vacuum.

Solution: Initially the two pistons are in equilibrium; hence, the pressure underneath them must be equal. Since there is vacuum above the pistons, the force of pressure acting on each piston must balance the weight of that piston. Thus, the piston having twice the mass must have twice the cross-sectional area. We can denote these two areas A and $2A$.

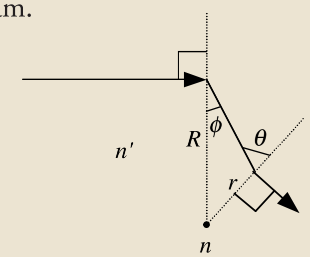
If we double the total load on the lighter piston, the extra load on the piston would cause it to drop all the way to the bottom of its cylinder and all of the gas will be squeezed into the second cylinder. Since there is negligible variation in hydrostatic pressure across the height of a 40-cm column in a typical gas under reasonable conditions, one can assume the pressure is the same everywhere throughout the gas. But the pressure in this second cylinder must continue to hold up the second piston, whose weight has not changed. Thus, the gas pressure will be the same before and after adding the extra 1-kg load. Furthermore, the temperature and the number of moles are the same. Thus, the volume of the gas must be conserved. The initial volume is $3Ah$. The final volume is $2AH$. Equating these two expressions gives $H = 1.5h = 0.60$ m.

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

The Ray of Hope

Challenge: A capillary tube is made of glass with the index of refraction n'. The outer radius of the tube is R. The tube is filled with a liquid with the index of refraction $n < n'$. What should be the minimum internal radius of the tube r so that any ray that hits the tube would enter the liquid?

Solution: The limiting case for penetration into the liquid occurs when the incident beam from air just grazes the outer surface of the tube tangentially and the beam transmitted into the liquid just grazes the inner surface of the tube tangentially, as sketched in the following diagram.



Applying Snell's law first at the outer surface, we have

$$\sin\phi = \frac{1}{n'}, \quad (1)$$

and then at the glass-liquid interface,

$$\sin\theta = \frac{n}{n'}. \qquad (2)$$

Next we can apply the law of sines to the triangle formed by the dashed sides labeled *R* and *r* and the ray propagating in the glass,

$$\frac{\sin\phi}{r} = \frac{\sin(\pi-\theta)}{R} = \frac{\sin\theta}{R}. \qquad (3)$$

Now we can substitute Eqs. (1) and (2) into Eq. (3) and simplify to obtain the minimum core radius of

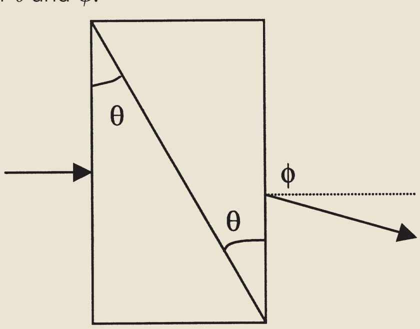
$$r = \frac{R}{n}.$$

As in many problems involving propagation through several media, only the refraction indices of the first (air) and last (liquid) media appear in the solution.

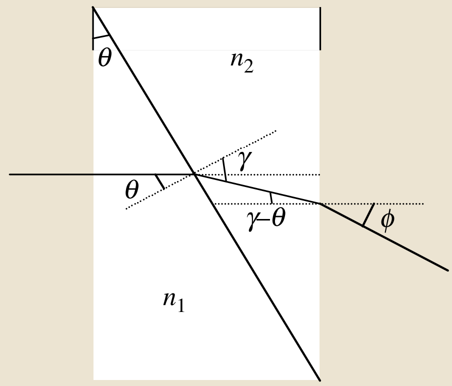
(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

Beam Me Up!

Challenge: Two identical prisms with slightly different indices are located as shown. Angle θ is small. When a laser beam strikes one of the prisms perpendicular to the surface, the refracted ray is deviated by a small angle ϕ. Find the difference *dn* between the indices of refraction of the prism in terms of θ and ϕ.



Solution: The internal angles of refraction are sketched in the following diagram.



Applying Snell's law at the diagonal interface between the two prisms gives

$$n_1 \sin\theta = n_2 \sin\gamma \Rightarrow n_1\theta \cong n_2\gamma \qquad (1)$$

for small angles. Next we can apply Snell's law at the exit vertical plane of the second prism to find

$$n_2 \sin(\gamma-\theta) = \sin\phi \Rightarrow n_2\gamma - n_2\theta \cong \phi, \qquad (2)$$

assuming transmission into air. Substituting Eq. (1) for the first term on the left-hand side of Eq. (2) gives

$$dn \equiv n_1 - n_2 = \frac{\phi}{\theta}.$$

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

Several other readers also sent us correct solutions to the March Challenges. We would like to recognize the following contributors:

James J. Carr (Webster, NY)

John F. Goehl, Jr. (Barry Univ., Miami Shores, FL)

Art Hovey (Milford, CT)

Allan T. Mense (University of Phoenix, Green Valley, AZ)

We appreciate your submissions and hope to receive more solutions in the future.

Note to contributors: As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– Please email the solutions as Word files;
– Please name the file "September03BSimpson" if—for instance—your name is Bart Simpson, and you are sending the solutions to September 2003 Challenges;
– Please state your name, hometown, and professional affiliation in the *file*, not only in the email message.

Many thanks!

Please send correspondence to:

Boris Korsunsky
444 Wellesley St.
Weston, MA 02493-2631
korsunbo@gse.harvard.edu