Physics Challenges for Teachers and Students

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Here we present the solutions to the March 2002 *Challenges:*

A Vacuum Squeezer

From the first law of thermodynamics,

$$dU = dQ - dW,$$

where dU is the change in the internal energy of the gas, dQ is the heat added to the gas, and dW is the work done by the gas.
For a monoatomic ideal gas

$$U = \tfrac{3}{2}nRT,$$

where n is the quantity of gas in moles, R is the universal gas constant, and T is the absolute temperature of the gas. For a fixed quantity of the gas

$$dU = \tfrac{3}{2}nR\,dT. \qquad (1)$$

The work done by the expansion of the gas is

$$dW = P\,dV.$$

If the cross-section area of the piston is A, the spring constant k, and the distance from the left edge of the container to the piston x, then the volume of gas in the cylinder is

$$V = Ax \quad \text{and} \quad dV = A\,dx.$$

The force on the piston exerted by the spring is kx; therefore, the pressure of the gas is

$$P = \frac{kx}{A} \quad \text{and} \quad dP = \frac{k\,dx}{A}.$$

From the ideal gas law:

$$PV = nRT \text{ or } d(PV) = d(nRT) \text{ or}$$
$$P\,dV + V\,dP = nR\,dT.$$

Since $P\,dV = V\,dP = kx\,dx$, the ideal gas law can be written as

$$2P\,dV = nR\,dT$$
or
$$dW = Pd\,V = (1/2)nRdT\,. \qquad (2)$$

Substituting for dU and dW in the first law using Eq. (1) and Eq. (2):

$$\tfrac{3}{2}nR\,dT = dQ - \tfrac{1}{2}nR\,dT \quad \text{or} \quad dQ = 2nR\,dT.$$

The heat capacity of the system is then

$$C = \frac{dQ}{dT} \quad \text{or} \quad C = 2nR.$$

With the quantity of gas $n = 1$ mol,

$$C = (2 \text{ mol})R \approx 16.6 \text{ J/K}.$$

(*Contributed by Richard Morra, Pascack Valley Regional HS District (Retired), Hillsdale, N.J.*)

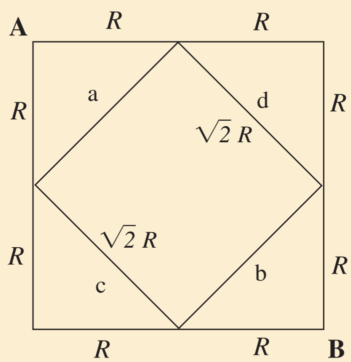
Futile resistance

Let V_1 be the voltage across the ammeter in the first circuit. In the second circuit, that voltage becomes $3V_1$ because the current triples. Similarly, the initial voltmeter reading can be denoted V_2, and the final reading – $V_2/3$. Since the battery remains in series with the ammeter, the battery current does not depend on the resistance distribution between the battery and the ammeter; for convenience, we can assume that the battery has zero resistance. Then the voltage across the battery remains 6 V, so:

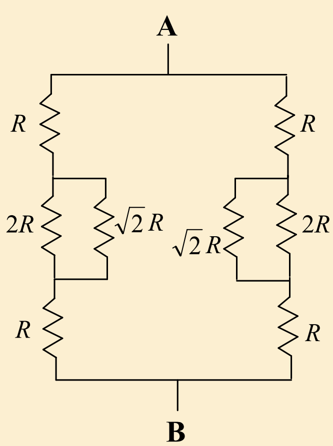
$$V_1 + V_2 = 6$$
$$3V_1 + (V_2/3) = 6,$$
$$\text{and } V_2 = 4.5 \text{ V}.$$

(*Contributed by Christina Domnisoru, a junior at Maine School of Science and Mathematics, Limestone, Maine*)

Red Square



By symmetry, wires a and b each have no potential difference between their ends. Thus, no current flows through these wires and they may be removed. Assuming that the half-side of the large square has a resistance of $R = \frac{\rho d}{2}$, the circuit reduces to the following:



The resistance of the left side is

$$2R + \frac{2R \cdot \sqrt{2}R}{2R + \sqrt{2}R} = 2R + \frac{2\sqrt{2}}{2 + \sqrt{2}}R = 2\sqrt{2}R.$$

Meanwhile, the right side has the same resistance, so the resistance between points A and B is $\sqrt{2}R = \frac{\sqrt{2}}{2}\rho d.$

(*Contributed by Gary Johnson, Michigan Lutheran Seminary, Saginaw, Mich.*)

Several other readers also submitted correct solutions for this issue. We wish to thank and recognize these contributors:

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