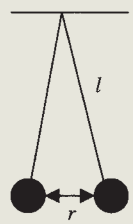
Physics Challenges for Teachers and Students

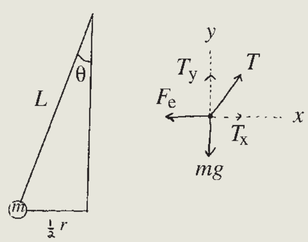
Solutions for February 2004 *Challenges*

A Slow Approach

Challenge: Two small balls of mass m each are suspended by light strings of length l as shown. Initially, each ball carries a positive charge q. The initial separation of the balls is r ($r << l$). The charge of each ball slowly drains into surroundings; the charge on each ball changes with time as $q_t = q(1 - bt)^{1.5}$, where b is a given constant. As a result, the balls get closer. Find the speed at which the balls approach each other.



Solution: Set up a force diagram for one of the balls:



The Coulomb force is

$$F_e = \frac{kq_1q_2}{r^2} = \frac{k[q(1-bt)^{1.5}][q(1-bt)^{1.5}]}{r^2}$$

$$F_e = \frac{kq^2(1-bt)^3}{2}.$$

The tension component in the horizontal direction is

$$T_x = T \sin \theta.$$

From the vertical component of the tension we have

$$T_y = mg$$

$$T \cos \theta = mg$$

$$T = \frac{mg}{\cos \theta}.$$

Thus, substituting above we have

$$T_x = T \sin \theta = \frac{mg}{\cos \theta} \sin \theta = mg \tan \theta.$$

Applying the trig definition for tangent we have

$$T_x = mg \tan \theta = mg \frac{\left(\frac{r}{2}\right)}{L} = \frac{mgr}{2L}.$$

Thus, we write Newton's second law in the horizontal direction as

$$\frac{mgr}{2L} - \frac{kq^2(1-bt)^3}{r^2} = ma_x.$$

If the charge is draining slowly (presumably b is small), we can approximate the system as being close to equilibrium, and a_x will be very small.

$$a_x \approx 0$$

Thus,

$$\frac{mgr}{2L} - \frac{kq^2(1-bt)^3}{r^2} \approx 0$$

$$\frac{mgr}{2L} = \frac{kq^2(1-bt)^3}{r^2}$$

$$r^3 = \frac{2Lkq^2\,(1-bt)^3}{mg}$$

$$r = \left(\frac{2Lkq^2\,(1-bt)^3}{mg}\right)^{1/3} = \left(\frac{2Lkq^2}{mg}\right)^{1/3}(1-bt).$$

Take the derivative to find the velocity

$$v = \frac{dr}{dt} = -b\left(\frac{2Lkq^2}{mg}\right)^{1/3}$$

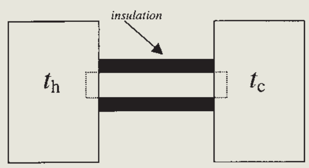
$$v = -\left(\frac{2Lkq^2b^3}{mg}\right)^{1/3}.$$

(Contributed by Kelly Krieble, Moravian College, Bethlehem, PA)

Fire and Ice

Challenge: An insulated container is filled with a mixture of water and ice at t_c = 0°C. Another container is filled with water that is continuously boiling at t_h = 100°C. In a series of experiments, the containers are connected by various thick rods that pass through the walls of the containers (see diagram). The rod is insulated in such a way that there is no heat loss to surroundings.

In experiment 1, a copper rod is used, and the ice melts in



T_1 = 20 min.

In experiment 2, a steel rod of the same cross section is used, and the ice melts in T_2 = 60 min. How long would it take to melt the ice if the two rods are used "in series"?

Solution: The rate at which heat is transferred by a single rod is given by the expression:

$$P = \frac{Q}{T} = kA\,\frac{t_h - t_c}{L} = C(t_h - t_c)$$ in which A is

the area, L is the length, and k is the thermal conductivity, the only factor different in the two experiments. Since $A\,(t_h - t_l)/L$ is a constant, I denote it with C. When you put two rods in series you would have:

$$P = \frac{A(t_h - t_c)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}\ .$$

For our case, $L_1 = L_2 = L$. So we have:

$$P = \frac{A(t_h - t_c)}{L}\left(\frac{1}{1/k_1 + 1/k_2}\right) = C\left(\frac{k1\,k2}{k1 + k2}\right) \Rightarrow$$

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}.$$

We have $T_1 = Q/P_1$, $T_2 = Q/P_2$, and $T = Q/P$:

$$T = \frac{Q}{P} = \frac{Q}{P_1} + \frac{Q}{P_2} = T_1 + T_2 = 80 \text{ min.}$$

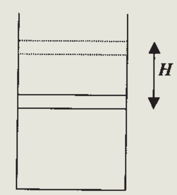
(Contributed by Amirali M. Shanechi, student, Don Mills C.I., Toronto, Canada)

Giving in to Pressure

Column Editor's note: A similar situation was discussed in C.E. Mungan's "Irreversible adiabatic compression of an ideal gas," *Phys. Teach.* **41**, 450–453 (Nov. 2003).

Challenge: A portion of helium gas in a vertical cylindrical container is in thermodynamic equilibrium with the surroundings. The gas is confined by a movable heavy piston. The piston is slowly elevated a distance H from its equilibrium position and then kept in the elevated position long enough for the thermodynamic equilibrium to be reestablished. After that, the container is insulated and then the piston is released. After the piston comes to rest, what is the new equilibrium position of the piston?

Solution: Label the initial equilibrium configuration of the system by subscript "1," the intermediate elevated configuration (in thermal equilibrium) by "2," and the final equilibrium configuration by "3." Denote the pressure, volume, temperature, and number of moles of helium gas by the symbols P, V, T, and n, respectively. In configuration 1, the pressure balance between the piston (of mass m and cross-sectional area A) and the helium gas implies that



$$P_1 = \frac{mg}{A} + P_{atm}, \tag{1}$$

including atmospheric pressure P_{atm} pushing down on the top of the piston. In both configurations 1 and 2, the gas is in thermal equilibrium with the surroundings at temperature T_0 say, so that

$$P_1 V_1 = nRT_0 = P_2 V_2 \tag{2}$$

from the ideal gas law, where R is the universal gas constant. Since the gas is insulated, no heat is transferred in or out of it during the transition between configurations 2 and 3. However, the atmosphere does external work on the system of the gas and piston, so that its energy E increases by

$$P_{atm} Ah = E_3 - E_2 + \tfrac{3}{2}P_3V_3 - \tfrac{3}{2}P_2V_2 - mgh \quad (3)$$

according to the first law of thermodynamics, assuming the piston moves downward a distance h between positions 2 and 3. The second equality follows from the facts that the internal energy of a monatomic gas like helium is $3PV/2$ and that the piston loses mgh of gravitational potential energy. In the end, the gas again balances the piston, so that $P_3 = P_1$ as given by Eq. (1). Also, the net change in volume of the gas between configurations 1 and 3 is given by $V_3 - V_1 = A(H - h)$ due to the upward and downward motion of the piston. Substituting these two identities into Eq. (3), together with Eqs. (1) and (2), and simplifying gives the final result

$$h = 0.6H, \quad (4)$$

which is equivalent to saying that the piston ends up $0.4H$ above its initial position 1.

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

• Several other readers also sent us correct solutions to the February *Challenges*. We would like to recognize the following contributors:

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We appreciate your submissions and hope to receive more solutions in the future.

• **Note to contributors:** As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– please email the solutions as Word files;
– please name the file "April04LSimpson" if — for instance — your name is Lisa Simpson, and you are sending the solutions to April 2004 Challenges;
– please state your name, hometown and professional affiliation in the file, not only in the email message.

Many thanks!

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