Physics Challenges for Teachers and Students

Solutions to the February Challenges

Shake and Bake

Challenge: An air-filled parallel-plate capacitor with the plate area A is connected to a battery with an emf E and small internal resistance. One of the plates vibrates so that the distance between the plates varies as $d = d_o + a \cos \omega t$ ($a << d_o$). The capacitor "bakes" (since the capacitor is *not* oil-filled, the conventional term "fries" is less appropriate!) when the instantaneous current in the circuit reaches the value of I. Find the maximum possible amplitude of vibrations a.

Solution: One can use the formula for the capacitance of a parallel-plate capacitor:

$$C = \varepsilon_o A/d.$$

According to the definition of capacitance, the charge on one plate is $Q = CE$, where E is the emf (and the voltage across the capacitor). Also, the current is given by

$$I(t) = dQ/dt$$

$$I(t) = E(dC/dt)$$

$$I(t) = E(dC/dd)(dd/dt)$$

One can see that $(dC/dd) = (-\varepsilon_o A/d^2)$.

Since $d \approx d_o$, $(dC/dd) \approx (-\varepsilon_o A/d_o^2)$.

Also, $(dd/dt) = (a\omega \sin \omega t)$

$$I(t) = (E\varepsilon_o A/d_o^2)(a\omega \sin \omega t)$$

The amplitude of the current is then

$I = E\varepsilon_o Aa\omega/d_o^2$, so

$$a = \frac{Id_o^2}{EA\omega\varepsilon_o}.$$

(Contributed by Art Hovey, Milford, CT)

Track and Field

Challenge: A metal rod of mass m can slide without friction along two parallel horizontal tracks connected by resistor R. A vertical magnetic field B exists in the region. The rod is given initial speed v by a quick push. What distance x will it cover before stopping? The distance between the tracks is l.

Solution: Using the expression for the force acting on a moving current-carrying wire in a magnetic field in conjunction with Newton's second law, we may write

$$F = m\, dv/dt = BIl,$$

where B is the strength of the field, I is the current and l is the length of the wire.
From Faraday's law and Ohm's law, we obtain that

$$I = \mathrm{E/R} = -Blv/R,$$

where E is the induced emf.
Therefore,

$$dv/dt = -B^2 l^2 v/mR,$$

where the negative sign indicates that v is decreasing. Thus, we can rewrite this as

$$v\, dt = -(mR/B^2 l^2)dv.$$

Since

$$x = \int v\, dt,$$

we can write

$$x = -(mR/B^2l^2)\int dv.$$

Inserting the proper limits on the integral (v for the lower and 0 for the upper), we can calculate the distance x the rod travels before stopping as

$$x = \frac{mvR}{B^2l^2}.$$

(Contributed by Terry T. Crow, retired, Mississippi State University, MS)

Current Affairs

Challenge: Two ammeters, 1 and 2, have different internal resistances: r_1 (known) and r_2 (unknown). Each ammeter has an analog scale such that the angular deviation of the needle from zero is proportional to the current. Initially, the ammeters are connected in series and then to a source of constant voltage. The deviations of the needles of the ammeters are θ_1 and θ_2, respectively. The ammeters are then connected in parallel and then to the same voltage source. This time, the deviations of the needles are θ_1' and θ_2', respectively. Find r_2.

Solution: Let the proportionalities in the ammeters be given by k, so that $\theta_i = k_i I_i$. Now, assuming equal currents in series, equal potential differences in parallel, and using Ohm's Law, we have:

Series: $(i)\ \theta_i = k_i \dfrac{V}{r_1+r_2}$ $\quad (ii)\ \theta_2 = k_2 \dfrac{V}{r_1+r_2}$

Parallel: $(iii)\ \theta_i' = k_1 \dfrac{V}{r_1}$ $\quad (iv)\ \theta_2' = k_2 \dfrac{V}{r_2}$

Dividing $[(i)/(iii)]/[(ii)/(iv)]$ and rearranging gives:

$$r_2 = r_1 \frac{\theta_2\theta_1'}{\theta_1\theta_1'}.$$

(Contributed by H. Scott Wiley, Weslaco, TX)

First to Submit Correct Solutions

Several other readers also sent us correct solutions to the February Challenges. We would like to recognize the following contributors:

James J. Carr (Webster, NY)
John F. Goehl Jr. (Barry University, Miami Shores, FL)
Boyko Kakaradov (student, Maine School of Science and Mathematics, Limestone, ME)
Richard N. Louie (Pacific Lutheran University, Tacoma, WA)
Carl E. Mungan (United States Naval Academy, Annapolis, MD)

We appreciate your submissions and hope to receive more solutions in the future.

Note to contributors:

As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

- Please email the solutions as Word files.
- Please name the file as : "May03HSimpson" if — for instance — your name is Homer Simpson, and you are sending the solutions to May 2003 Challenges.
- State your name, hometown, and professional affiliation in the *file*, not only in the email message.

Many thanks!

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