Physics Challenges for Teachers and Students

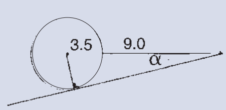
Solutions to December 2003 Challenges

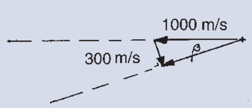
Rocket From the Pocket

(**Column Editor's Note:** *We do appreciate this reader's concern for the well-being of the commander and the alternative "friendly" solution. However, everyone should know that the commander happens to be a very tough space alien who can easily survive the accelerations involved…*)

Challenge: A spaceship is traveling in outer space at a constant velocity v =1000 m/s. Suddenly, the commander observes an asteroid straight ahead. The distance to the asteroid is L = 9.0 km; the diameter of the asteroid is D = 7.0 km. The commander attempts to take an evasive maneuver by firing the emergency rocket. The rocket gives the spaceship a very quick change in velocity δv = 300 m/s. Is it possible for the spaceship to avoid the collision? Assume that the commander can fire the rocket in any direction.

Solution: I suppose that L = 9.0 km is the distance **to the surface** of the asteroid.





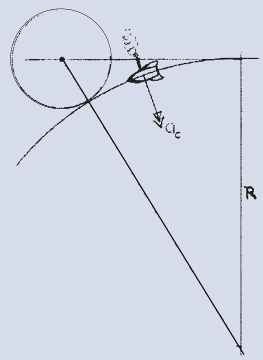
In Fig. 1 the angle that is needed to avoid a collision, is α.

$\sin(\alpha) = 3.5/12{,}5 = 0.28 \qquad \alpha = 16.3°.$

In Fig. 2 the maximum angle of deflection that is possible is β.

$\sin(\beta) = 300/1000 = 0.3 \qquad \beta = 17.5°.$

Clearly, $\alpha < \beta$: There is no collision.
The only problem is that the commander experiences a change in velocity of 300 m/s in a short period of time. The commander experiences an acceleration of at least 300 m/s^2. I have no idea if he/she/it can survive this acceleration…



A friendly solution is a circular trajectory with a radius R for the spaceship (that would require a continuous thrust, however).

$(R + 3{,}5)^2 = 12.5^2 + R^2.$

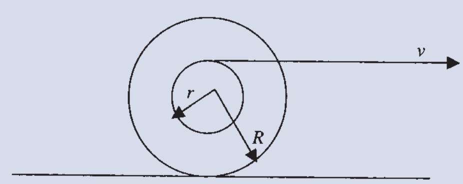
$R = 20{,}6$ km.

The centripetal acceleration is then $a_c = v^2/R =$ 48,6 m/s^2.
This gives the commander a better chance of surviving this adventure.

(Contributed by Leo H. van den Raadt, Heemstede, The Netherlands)

Spinning the Reel

Challenge: A reel is being pulled along a horizontal surface by the string wrapped around as shown. There is no slipping between the reel and the surface. The string is pulled at a constant speed v. Find the angular speed v of the reel assuming that r and R are given.



Solution: The fastest solution is to note that the speed v of the string equals the speed of the top-most point of the inner diameter of the reel. But this point is a distance $R + r$ above the bottom-most point of the outer diameter. Since this latter is the point at which the reel contacts the floor without slipping, it is instantaneously at rest. Thus the spool instantaneously rotates about that contact point with angular speed $\omega = v/(R + r)$.

A slightly longer, alternative solution is often helpful for introductory students who are not comfortable with the concept of instantaneous axes of pure rotation. Suppose the reel rolls forward one revolution. This means the entire reel in general, and the top-most point of the inner diameter in particular, has moved forward a distance of $2\pi R$. During this time, we have unreeled an additional length $2\pi r$ of string. Thus the total displacement of any initially unreeled point on the string is $2\pi R + 2\pi r$, during a time interval of one period of revolution T. That is, the speed of this fixed point on the unreeled string is $v = 2\pi(R + r)/T$. However, the angular speed of rotation of the reel is $\omega = 2\pi/T$ and so we once again conclude that $\omega = v/(R + r)$.

(Contributed by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

High Jump

Challenge: A student of height h jumps vertically up from the "squat" position. At the top point of the jump, the student's center of mass is at a height $3h/4$ from the ground. Find the average force F acting on the floor prior to the moment when the student loses contact with the floor. It is known that when the student stands on the floor, the center of mass is at a height $h/2$ from the floor; in the "squat" position, the center of mass is at a height $h/4$ from the floor. The mass of the student is m.

Solution: **Method 1:** Even though the normal force with the floor does no physical work on the person (energy transfers occur internally from the muscles), one can write a mathematically correct expression as though the contact force with the floor does change the person's energy. Hence, we can treat the center of mass motion as $W_{net} = \Delta K_{cm}$. Here, we are treating it as though only gravity and the floor act on the person, and since the person is in contact with the floor until the center of mass is a position $h/2$ from the ground, then

$$F_{floor}\frac{h}{4} + \left(mg\right)\left(\frac{-h}{2}\right) = 0$$

as the student starts and ends this motion at rest. Therefore, $F_{floor} = 2mg$.

Method 2: Since the feet come off the floor at a position $h/2$ from the ground, then the person accelerates upward from $h/4$ to $h/2$ with the same magnitude as the acceleration from $h/2$ to $3h/4$ by symmetry. Therefore, the net force on the student going upward is the same in magnitude as the student in free fall from $h/2$ to $3h/4$. Hence, while in contact with the floor, using Newton's second law,

$$F_{floor} - mg = mg \Rightarrow F_{floor} = 2mg.$$

(Contributed by Michael C. Faleski, Delta College, Midland, MI)

Several other readers also sent us correct solutions to the December *Challenges*. We would like to recognize the following contributors:

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We appreciate your submissions and hope to receive more solutions in the future.

Note to Contributors:

As the number of submissions grows, we request that certain guidelines be observed, in order to facilitate the process more efficiently:

– Please email the solutions as Word files;
– Please name the file as “Feb04LSimpson” if — for instance — your name is Lisa Simpson, and you are sending the solutions to February, 2004 *Challenges;*
– Please state your name, hometown and professional affiliation in the file, not only in the email message.

Many thanks!

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